

## MCAS Open-Response Questions & Answers

### Mathematics -- High School Level

#### Question # 1

- (A) A cube has side length of 3 cm. The cube is filled with grains of rice. If a larger cube has side length that is twice that of the first cube, how much more rice will the larger cube hold than is contained in the first cube?
- (B) A soup can has a paper label around it that measures 176 sq. cm. The radius of the can is 4 cm. Find the height of the can (use  $\pi = 22/7$ ).
- (C) The radius of a larger can is twice as much as the can in #2 above but the height remains the same. How much more paper will be needed for the label around this second can?



#### Answers

#### Question # 1

- (A) Volume of the cube =  $(3)(3)(3) = 27 \text{ cm}^3$ .  
The volume of the second cube is  $(6)(6)(6) = 216 \text{ cm}^3$ .  
The larger cube will hold  $189 \text{ cm}^3$  more grains of rice than the smaller cube ( $216 - 27 = 189$ ).
- (B) Lateral area (soup can label) =  $2(22/7)(4)(\text{height})$ .  
Setting this quantity = 176 gives a height of 7 cm.
- (C) New radius = 8 cm; height is still 7 cm.  
Lateral area =  $2(22/7)(8)(7) = 352 \text{ sq. cm}$ .  
This means you will need twice as much paper, or an additional 176 sq. cm.



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### Question #2

John is planning to create a garden in his backyard. He is using graph paper to lay out the design of his garden. The distance between adjacent coordinate points represents 1 foot. His garden had the following coordinates: A (-3,4), B (-4,-3), C (8, -3), and D (4,4).

- (A) Using the distance formula, calculate the number of feet of fencing that John will need to fence around the entire garden. (Round up to nearest foot.)
- (B) John wants to put a scarecrow at the midpoint of the diagonal AC. What is the coordinate of that point?
- (C) Will a scarecrow placed at the midpoint of diagonal BD be located at the same point as the scarecrow in Part B? If not at the same point, how far apart will the scarecrows be?



### Answers Question #2

- (A) First, draw out the figure of the garden and it is a trapezoid.  
Calculated distances are:  
AB ~ 7.1 ft.  
BC ~ 12 ft  
CD ~ 8.1 ft.  
DA ~ 7 ft.  
Sum = 34.2 ft., rounded up to 35 ft.
- (B) Midpoint of AC is  $(5/2, 1/2)$ .
- (C) Midpoint of BD is  $(0, 1/2)$ , so they are not at the same location.  
The second scarecrow will be about 2.5 ft. away.

MCAS Open-Response Questions & Answers  
Mathematics -- High School Level

Question #3

Given a triangle ABC with the coordinates at A (-7, 3), B (-4, 6) and C (-1, 1).

- (A) Give the coordinates of triangle A'B'C' if triangle ABC is reflected over:
- 1) the x-axis
  - 2) the y-axis
  - 3) the line  $y = x$
- (B) Give the coordinates of triangle A'B'C' that is formed by translating triangle ABC 4 units to the right and 8 units down.
- (C) Give the coordinates of triangle A'B'C' that is formed by rotating triangle ABC 180 degrees using the origin, (0,0) as the center of rotation.

 Answers  
Question #3

- (A) 1)  $A' = (-7, -3)$ ,  $B' = (-4, -6)$ ,  $C' = (-1, -1)$   
2)  $A' = (7, 3)$ ,  $B' = (4, 6)$ ,  $C' = (1, 1)$   
3)  $A' = (3, -7)$ ,  $B' = (6, -4)$ ,  $C' = (1, -1)$
- (B)  $A' = (-3, -5)$ ,  $B' = (0, 2)$ ,  $C' = (3, -7)$
- (C)  $A' = (7, -3)$ ,  $B' = (4, -6)$ ,  $C' = (1, -1)$

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question #4

Jan noticed a soda machine at a distance of 24 ft. and started to walk toward the machine at a constant rate. It took her 12 seconds to reach the machine.

- (A) Draw a graph to show Jan's travel.
  - i) What are units for the x-axis?
  - ii) What are the units for the y-axis?
- (B) How far had Jan traveled in the first 4 seconds?
- (C) How far from the soda machine was she after 8 seconds?
- (D) Write an equation in the slope intercept form to predict the distance from the machine as a function of time. Why is the slope negative?
- (E) Jan walked at a steady pace. What was her rate (speed)?  
Was this a fast or slow pace and explain your answer.



### Answers

### Question #4

- (A) i) Time in seconds is on the x-axis,
  - ii) Distance from machine in feet is on the y-axis.
- (B) 8 ft.
- (C) 8 ft.
- (D)  $y = -2x + 24$ ; the negative sign indicates the distance from the soda machine is decreasing as time increases.
- (E) Rate = 2 ft./ sec. This is very slow.  
A person normally walks 5 – 6 feet per second.

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question #5

A student reads the following passage from a novel:

*“Simon sat next to an elderly man during his train ride to Grand Central Station. The two men sat side by side for the entire 200-mile trip. Over that distance Simon spoke about forty thousand words to the stranger. When the train stopped at Grand Central Station, the two men departed and they never saw each other again.”*

- (1) Determine a reasonable number for the rate, in words per minute, or normal spoken language. Show your calculations and explain your reasoning.
- (2) Make an estimate of the average speed of a train in miles per hour.
- (3) Determine whether the statement – “Simon spoke about forty thousand words to the stranger” is reasonable. Why is it or why is it not reasonable? Show your calculations and explain your reasoning.



### Answers Question #5

- (1) There are about 60 words in the passage cited. Reading that aloud (or to oneself) takes about 20 seconds.  $60 \text{ words}/20 \text{ sec} = 180 \text{ wpm}$ .
- (2) A train might go about 60 mph (some go faster, but some make stops).
- (3) At a rate of 60 mi/hr, traveling 200 miles takes  $3 \frac{1}{3}$  hours, or 200 minutes (alternately,  $60 \text{ mi/hr} = 1 \text{ mi/min}$  so 200 miles takes 200 minutes).
- (4) At a rate of 180 words/min, the number of words in 200 min =  $(180)(200) = 36,000$  words, assuming Simon spoke all the time. This is not quite 40,000 words, but is possible, if the person talked faster, or the train went slower, or made several stops.

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### Question #6

- (1) Simplify the expression:  $24 - 12 / 6 * 2^3$
- (2) Simplify:  $[ 30 - ( 15 / 3 ) + 8 ] * 4$
- (3) The operation  $\#$  is defined as  $A \# B = 3A - 2B$ ,  
where A and B are real numbers.  
Use this definition to find the value of  $G \# H$ , if  $G = 6$  and  $H = -3$ .
- (4) A rectangle named MATH is drawn on a coordinate system so that the vertices are at  $M = (-2, 3)$ ,  $A = (-2, 19)$ ,  $T = (10, 19)$  and  $H = (10, 3)$ .  
Diagonals MT and AH are drawn.
- Find the slope of the diagonal MT.
  - Find the midpoint of the diagonal MT.
  - Show that the diagonal AH has the same midpoint as diagonal MT.
  - Show that the lengths of the diagonals are the same, e.g., that  $MT = AH$ .



### Answers Question #6

$$\begin{aligned} (1) \quad 24 - 12 / 6 * 2^3 &= 24 - 12 / 6 * 8 \\ &= 24 - 2 * 8 \\ &= 24 - 16 \\ &= 8. \end{aligned}$$

$$\begin{aligned} (2) \quad [ 30 - (15 / 3) + 8 ] * 4 &= [ 30 - 5 + 8 ] * 4 \\ &= [ 25 + 8 ] * 4 \\ &= 33 * 4 \\ &= 132. \end{aligned}$$

## MCAS Open-Response Questions & Answers

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#### Question 6 continued

(3)  $G \# H = 6 \# (-3) = 3(6) - 2(-3) = 18 + 6 = 24.$

(4) (Picture would be helpful in solution, but not required)

a) slope of MT  $= (2^{\text{nd}} y - 1^{\text{st}} y)/(2^{\text{nd}} x - 1^{\text{st}} x)$   
 $= (19 - 3)/(10 - (-2))$   
 $= 16/12$   
 $= 4/3.$

b) midpoint of MT  $= (\text{average of } x\text{'s}, \text{average of } y\text{'s})$   
 $= ((-2 + 10)/2, (3 + 19)/2)$   
 $= (4, 11).$

c) midpoint of AH  $= ((-2 + 10)/2, (19 + 3)/2)$   
 $= (4, 11)$

This is the same point as the midpoint of MT.

d) Use the distance formula:

$$d = \text{sq. root of } [(2^{\text{nd}} y - 1^{\text{st}} y)^2 + (2^{\text{nd}} x - 1^{\text{st}} x)^2]$$

For MT:  $\text{sq root of } [(19-3)^2 + (10 - (-2))^2]$

$$= \text{sq root of } [16^2 + 12^2]$$

$$= \text{sq root of } [256 + 144]$$

$$= \text{sq root of } 400$$

$$= 20.$$

For AH:  $\text{sq root of } [(3 - 19)^2 + (10 - (-2))^2]$

$$= \text{sq root of } [(-16)^2 + 12^2]$$

$$= \text{sq root of } [256 + 144]$$

$$= \text{sq root of } 400$$

$$= 20.$$

Since both distances are equal,  $MT = AH.$

# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level

### Question #7

Solve the quadratic equation  $x^2 - 4x = 12$  in 3 ways – showing all work for each method. Verify that you get the same answer by the methods of factoring, completing the square, and using the quadratic formula.



### Answers Question #7

(1) Factoring:

$$\begin{aligned}x^2 - 4x &= 12 \\x^2 - 4x - 12 &= 0 \\(x - 6)(x + 2) &= 0 \\x - 6 = 0 \text{ or } x + 2 = 0 \\x &= 6 \text{ or } x = -2.\end{aligned}$$

(2) Completing the square:

$$\begin{aligned}x^2 - 4x &= 12 \\x^2 - 4x + 4 &= 12 + 4 \\(x - 2)^2 &= 16 \\x - 2 = 4 \text{ or } x - 2 = -4 \\x &= 6, \text{ or} \\x &= -2.\end{aligned}$$

(3) Quadratic Formula:

$$\begin{aligned}x^2 - 4x &= 12 \\x^2 - 4x - 12 &= 0, \\ \text{so in the formula } ax^2 + bx + c &= 0 \\ \text{you have } a = 1, b = -4, c &= -12.\end{aligned}$$

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 7:3 continued*

$$\begin{aligned}\text{Quadratic Formula: } x &= [-b \pm \sqrt{b^2 - 4ac}]/(2a) \\ &= [-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}]/(2 \cdot 1) \\ &= [4 \pm \sqrt{16 + 48}]/2 \\ &= [4 \pm \sqrt{64}]/2 \\ &= [4 \pm 8]/2 \\ &= (4 + 8)/2 \text{ or } (4 - 8)/2 \\ &= 12/2 \text{ or } -4/2 \\ &= 6 \text{ or } -2.\end{aligned}$$

Therefore, all 3 methods yield the same solution to the original problem,  
that is,  $x = 6$  or  $x = -2$ .



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### Question #8

- (1) Add:  $(5x^2 - 3x + 10) + (2x^2 + 9x - 6)$
- (2) Subtract:  $(7a^3 + 4a^2 - 5a - 8) - (6a^3 + 2a^2 - a - 13)$
- (3) Multiply:  $2n^2(5n^2 - 3n + 4)$
- (4) Multiply:  $(3x + 5)(2x - 4)$
- (5) Divide:  $(12x^3 - 9x^2 + 15x - 18) / (3x)$
- (6) Find the perimeter of a square whose side is  $3x + 9$ .
- (7) Find the area of a rectangle whose width is  $2x + 4$  and length is  $8x - 3$ .
- (8) Find the length of a rectangle whose area is  $30x^2 + 18x$  and whose width is  $6x$ .



### Answers Question #8

- (1)  $7x^2 + 6x + 4$
- (2)  $a^3 + 2a^2 - 4a + 5$
- (3)  $10n^4 - 6n^3 + 8n^2$
- (4)  $6x^2 - 2x - 20$
- (5)  $4x^2 - 3x + 5 - 6/x$
- (6)  $12x + 36$
- (7)  $16x^2 + 26x - 12$
- (8)  $5x + 3$



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#### Question #9

Years of post-secondary education	1	2	2.5	3	4	6	6.5	7.5
Entry-level salary (in thousands of \$\$)	17	19	24	20	27	35	40	45

- Plot the points and make a scatter plot to represent the data in the chart. Be sure to label the axis appropriately.
- Draw your best approximation of the line of best fit.
- Write an algebraic equation of the line of best fit.
- Could the line of best fit lead to an unreasonable conclusion about the relationship between years of education and salaries? Justify your answer.



#### Answers

#### Question #9

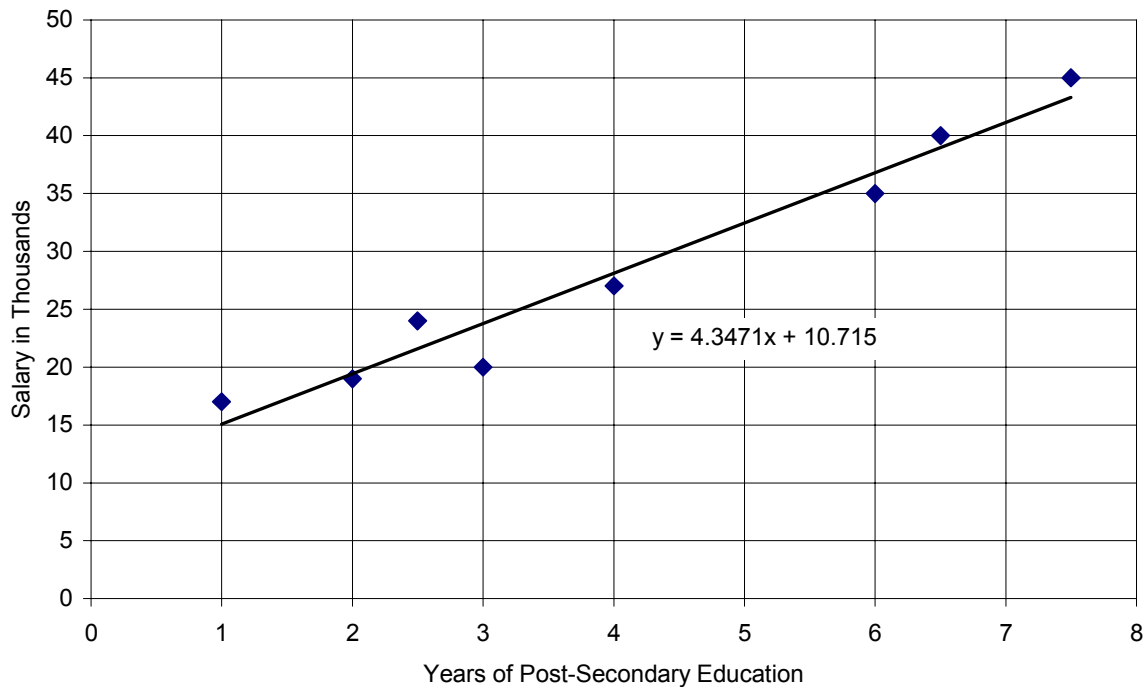
- Plot the points and make a scatter plot to represent the data in the chart. Be sure to label the axis appropriately.
- Draw your best approximation of the line of best fit.
- Write an algebraic equation of the line of best fit.



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 9:d continued

Education vs. Salary



- d) Could the line of best fit lead to an unreasonable conclusion about the relationship between years of education and salaries? Justify your answer.

*Yes. The line of best fit leads makes a direct correlation between the years of secondary education and salary where none necessarily exists. Additionally, it shows the relationship to continue indefinitely when conventional logic suggests that it will level off eventually.*



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### Question #10

The following table represents the daily closing prices for AOL Time Warner from November 1, 2002 through November 29, 2002.

Date	Closing Price
11/1/02	15.15
11/4/02	15.53
11/5/02	15.77
11/6/02	16.29
11/7/02	15.36
11/8/02	15.36
11/11/02	14.95
11/12/02	15.06
11/13/02	15.22
11/14/02	15.30
11/15/02	15.42
11/18/02	15.14
11/19/02	14.97
11/20/02	14.80
11/21/02	15.65
11/22/02	15.89
11/25/02	16.21
11/26/02	15.36
11/27/02	16.40
11/29/02	16.37

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 10 continued*

Make a scatterplot using the values for the closing quote (price) of AOL Time Warner then answer the following questions:

- 1) Calculate the slope between November 11<sup>th</sup> and November 15<sup>th</sup>.
- 2) Relative to the value of the stock, what real-world meaning does the slope have? (Refer back to the graph. Connect the points from November 11<sup>th</sup> through November 15<sup>th</sup> if necessary.)
- 3) What real-world meaning does the sign of the slope have?
- 4) Calculate the slope between November 15<sup>th</sup> and November 20<sup>th</sup>.
- 5) Relative to the value of the stock, what real-world meaning does the slope have? (Refer back to the graph. Connect the points from November 15<sup>th</sup> through November 20<sup>th</sup> if necessary.)
- 6) Are there any places on the graph where the slope is zero? How do you recognize them? What do they mean?
- 7) There are no places on the graph where the slope is undefined. Why not? What would an undefined slope mean relative to the value of the stock?



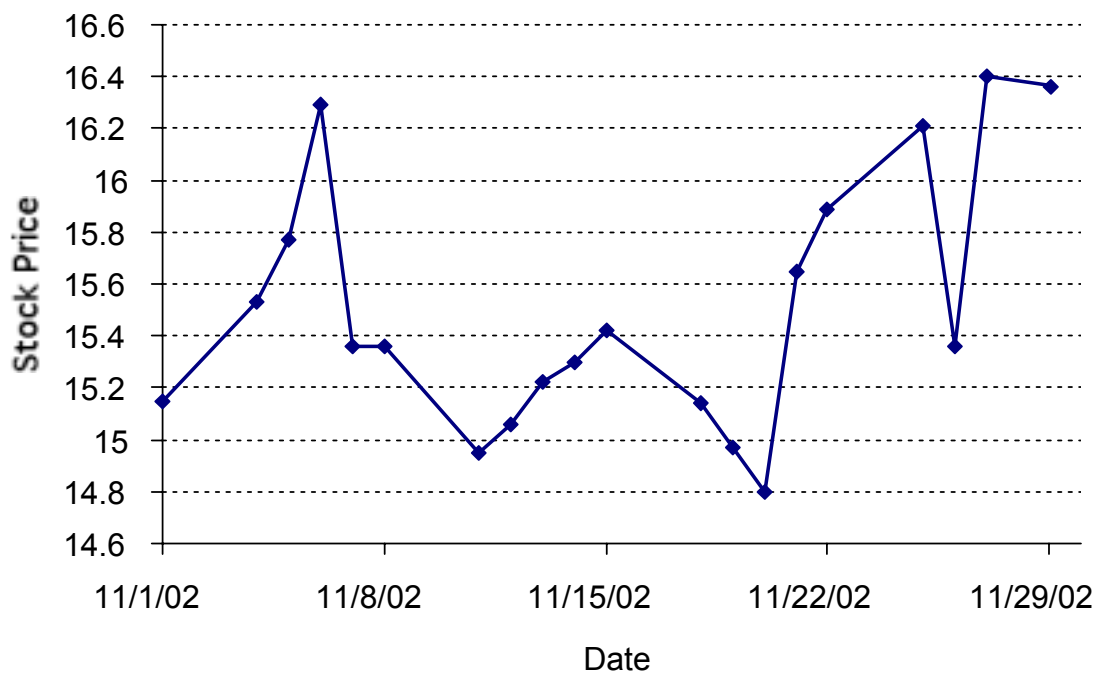
# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level



### Answers Question 10

AOL Time Warner



Make a scatterplot using the values for the closing quote (price) of AOL Time Warner then answer the following questions:

- 1) Calculate the slope between November 11<sup>th</sup> and November 15<sup>th</sup>.

$$\text{slope} = \frac{\text{change in price}}{\text{change in time}} = \frac{15.42 - 14.95}{15 - 11} = \frac{0.47}{4} = 0.1175$$

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 10 continued

- 2) Relative to the value of the stock, what real-world meaning does the slope have? (Refer back to the graph. Connect the points from November 11<sup>th</sup> through November 15<sup>th</sup> if necessary.)

*The average change in price per day from November 11<sup>th</sup> through the 15<sup>th</sup>.  
In this case it means that the stock price changed \$0.1175 per day.*

- 3) What real-world meaning does the sign of the slope have?

*Whether the price of the stock was increasing or decreasing over the  
given time interval.*

- 4) Calculate the slope between November 15<sup>th</sup> and November 20<sup>th</sup>.

$$\text{slope} = \frac{\text{change in price}}{\text{change in time}} = \frac{14.8 - 15.42}{20 - 15} = \frac{-0.62}{5} = -0.124$$

- 5) Relative to the value of the stock, what real-world meaning does the slope have? (Refer back to the graph. Connect the points from November 15<sup>th</sup> through November 20<sup>th</sup> if necessary.)

*The average change in price per day from November 15<sup>th</sup> through the 20<sup>th</sup>.  
In this case it means that the stock price changed \$-0.124 per day  
because the slope is negative it indicates that the price dropped.*



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 10 continued*

- 6) Are there any places on the graph where the slope is zero? How do you recognize them? What do they mean?

*Yes, from November 7<sup>th</sup> through the 8<sup>th</sup>. They can be recognized by the horizontal line connecting those two stock values. It means that the stock did not change price over the given interval.*

- 7) There are no places on the graph where the slope is undefined. Why not? What would an undefined slope mean relative to the value of the stock?

*No. An undefined slope would be represented by a vertical line on the graph. If a vertical line were possible on the above graph, it would indicate that the stock had two prices simultaneously on the same day, which is impossible.*



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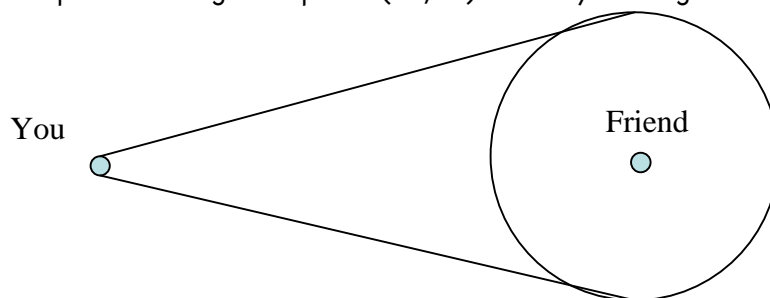
### Question # 11

A crop circle is a pattern impressed in a field of grain, snow, soil, and even stands of trees. The designs are usually comprised of circles. The crop or other medium is laid down in spiral patterns, sometimes woven. Features include circles, rings, spirals, mathematical representations, straight lines, triangles, and bars.

Stalks are bent over 90 degrees, unbroken. Often, crops continue to grow, but do not stand up again. It has been demonstrated that seeds from wheat within crop circles grow faster than wheat growing outside the crop circle area.

You and your friend stop to investigate a crop circle. You are standing at the point  $(9, 8)$ , and your friend is at  $(-9, -8)$ . The circle is centered around a marker at  $(0, 0)$ .

- 1) Find the distance between you and the marker.
- 2) Find the distance between you and your friend.
- 3) Write the standard form of the equation of the crop circle.
- 4) Write an equation of the line that is tangent to the crop circle and passes through the point  $(-9, 8)$  where your dog is located.



- 5) Does the tangent line in *Exercise 4* intersect the car that is located at the point  $(-15, 4)$ ?
- 6) When leaving, you and your friend notice a larger crop circle that can be modeled by the equation  $y^2 = 3844 - x^2$ , where  $x$  and  $y$  are measured in feet. Graph the equation. Assume the center is at  $(0,0)$ .

## MCAS Open-Response Questions & Answers Mathematics -- High School Level



### Answers Question #11

- 1) Find the distance between you and the marker.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(9 - 0)^2 + (8 - 0)^2} \\&= \sqrt{9^2 + 8^2} \\&= \sqrt{81 + 64} \\&= \sqrt{145}\end{aligned}$$

- 2) Find the distance between you and your friend.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(9 - -9)^2 + (8 - -8)^2} \\&= \sqrt{18^2 + 16^2} \\&= \sqrt{324 + 256} \\&= \sqrt{580} \\&= \sqrt{145}\end{aligned}$$

*Additionally, you and your friend are the same distance from the marker; therefore, the distance from you to him is twice the distance from you to the marker.*

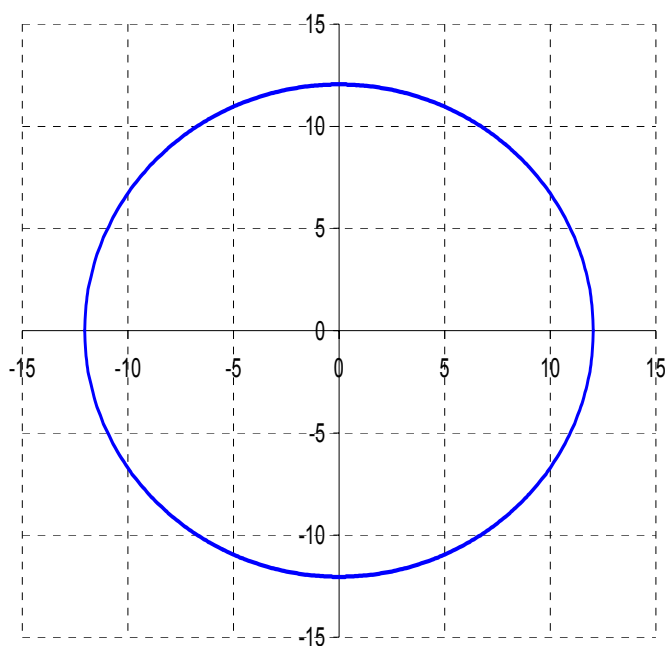


## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 11 continued

- 3) Write the standard form of the equation of the crop circle.

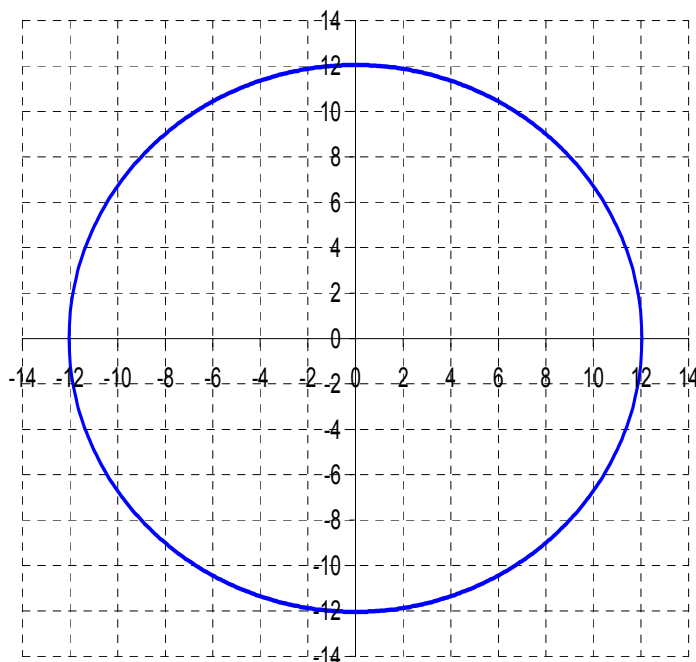
$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2 \\(x - 0)^2 + (y - 0)^2 &= (\sqrt{145})^2 \\x^2 + y^2 &= 145\end{aligned}$$



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 11 continued

- 4) Write an equation of the line that is tangent to the crop circle and passes through the point  $(-9, 8)$  where your dog is located.



Slope between the origin and the point  $(-9, 8)$  is:

$$m = \frac{y^2 - y^1}{x^2 - x^1} = \frac{8 - 0}{-9 - 0} = -\frac{8}{9}$$

Tangent lines are perpendicular to the point of tangency, therefore the slope of the line is:

$$m = \frac{9}{8}$$

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 11:4 continued*

► Through the point  $(-9, 8)$  ...

$$y - y^1 = m(x - x_1)$$

$$y - 8 = \frac{9}{8}(x - -9)$$

$$y - 8 = \frac{9}{8}x + \frac{81}{8}$$

$$y = \frac{9}{8}x + \frac{145}{8}$$



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question #12

A total of \$12,000 is invested in two funds paying 9% and 11% simple interest. If the yearly interest is \$1,180, how much of the \$12,000 is invested at each rate?



### Answers Question #12

A system of equations is a collection of two or more equations with the same set of unknowns. In solving a system of equations, we try to find values for each of the unknowns that will satisfy every equation in the system.

- ▶ We have two unknowns: the amount of money invested at 9% and the amount of money invested at 11%. Our objective is to find these two numbers.
- ▶ Sentence (1) "A total of \$12,000 is invested in two funds paying 9% and 11% simple interest." can be restated as:  
(The amount of money invested at 9%) +  
(The amount of money invested at 11%) = \$12,000.
- ▶ Sentence (2) "If the yearly interest is \$1,180, how much of the \$12,000 is invested at each rate?" can be restated as:  
(the amount invested at 9%) x 9% +  
(amount invested at 11% x 11%) = total interest of \$1,180.

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 12 continued*

- ▶ Since the amount of money invested at the two percentages are what we're looking for, those are the variables. Let's call them  $x$  and  $y$  respectively:

$$\begin{aligned}x &= \text{amount invested at } .9\% \\y &= \text{amount invested at } 11\%\end{aligned}$$

- ▶ The system therefore becomes:

$$\begin{aligned}x + y &= 1200 \\0.09x + 0.11y &= 1180\end{aligned}$$

**The system can be solved four different ways:**

- 1) Substitution: The method of substitution involves five steps:**

**Step 1:** Solve for  $y$  in equation (1).

$$\begin{aligned}x + y &= 12000 \\y &= 12000 - x\end{aligned}$$

**Step 2:** Substitute this value for  $y$  in equation (2). This will change equation (2) to an equation with just one variable,  $x$ .

$$\begin{aligned}0.09x + 0.11y &= 1180 \\0.09x + 0.11(12000 - x) &= 1180\end{aligned}$$

**Step 3:** Solve for  $x$  in the translated equation (2).

$$\begin{aligned}0.09x + 0.11(12000 - x) &= 1180 \\0.09x + 1320 - 0.11x &= 1180 \\1320 - 0.02x &= -140 \\-0.02x &= -140 \\x &= 7000\end{aligned}$$

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 12:1 continued

**Step 4:** Substitute this value of  $x$  in the  $y$  equation you obtained in Step 1.

$$\begin{aligned}x + y &= 12000 \\7000 + y &= 12000 \\y &= 5000\end{aligned}$$

**Step 5:** Check your answers by substituting the values of  $x$  and  $y$  into the other equation.

$$\begin{aligned}0.09x + 0.11 &= 1180 \\0.09(7000) + 0.11(5000) &= 1180 \\630 + 550 &= 1180 \\1180 &= 1180\end{aligned}$$

### 2) Elimination: The process of elimination involves four steps:

In a two-variable problem rewrite the equations so that when the equations are added, one of the variables is eliminated, and then solve for the remaining variable.

**Step 1:** Change the first equation by multiplying equation by  $-0.09$  to obtain a new and equivalent equation.

$$\begin{aligned}x + y &= 12000 \\-0.09x - 0.09y &= -1080\end{aligned}$$

**Step 2:** Add the new equation to the second equation.

$$\begin{aligned}-0.09x - 0.09y &= -1080 \\0.09x + 0.11y &= 1180 \\0.02y &= 100 \\y &= 5000\end{aligned}$$

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 12:2 continued

**Step 3:** Substitute  $y = 5000$  into the original first equation and solve for  $x$ .

$$\begin{aligned}x + y &= 12000 \\x + 5000 &= 12000 \\x &= 7000\end{aligned}$$

**Step 4:** Check your answers in the second equation (2).

$$\begin{aligned}0.09x + 0.11y &= 1180 \\0.09(7000) + 0.11(5000) &= 1180 \\630 + 550 &= 1180 \\1180 &= 1180\end{aligned}$$

**3) Matrices:** This method is essentially a shortcut for the method of elimination.

- ▶ Rewrite the equations without the variables and operators. The left column contains the coefficients of the  $x$ 's, the middle column contains the coefficients of the  $y$ 's, and the right column contains the constants.

$$\left[ \begin{array}{cc|c} 1 & 1 & 12000 \\ 0.09 & 0.11 & 1180 \end{array} \right]$$

- ▶ The objective is to reorganize the original matrix into one that looks like

$$\left[ \begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array} \right]$$

where  $a$  and  $b$  are the solutions to the system.

*The easiest way to accomplish this is with the TI-83 calculator, or equivalent.*



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 12:3 continued

**Step 1:** Enter the Matrix in the calculator

- ▶ Hit **MATRIX**
- ▶ Go across to **EDIT**
- ▶ Enter the dimensions of the matrix, in this case 2 x 3
- ▶ Enter the elements of the matrix as shown above
- ▶ Once done, hit **2<sup>ND</sup> QUIT**

**Step 2:** Solve the Matrix

- ▶ Hit **MATRIX**
- ▶ Go across to **MATH**
- ▶ Choose option **B: rref(** and hit enter
- ▶ Hit **MATRIX**
- ▶ Choose matrix **A** and hit enter
- ▶ You should now see **rref([A])** on the calculator screen
- ▶ Hit enter again and it will solve the matrix.

$$\left[ \begin{array}{cc|c} 1 & 0 & 7000 \\ 0 & 1 & 5000 \end{array} \right]$$

You can read the answers off the matrix as  $x = \$7,000$  and  $y = \$5,000$ .



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

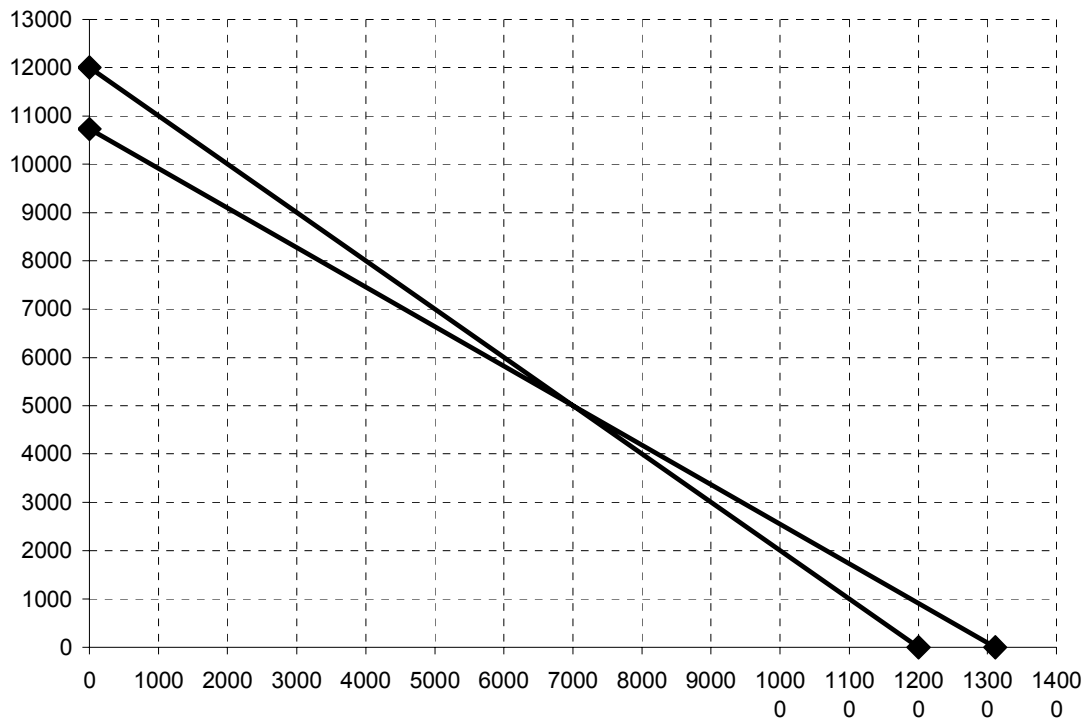
### Question 12:3 continued

#### 4) Graphing:

- ▶ In this method solve for  $y$  in each equation and graph both.
- ▶ The point of intersection is the solution.
- ▶ Alternatively, solve each equation for its  $x$  and  $y$  intercepts then graph:

$x + y = 12000$  has intercepts at  $(0, 12000)$  and  $(12000, 0)$

$0.09x + 0.11y = 1180$  has intercepts at  $(0, 10727)$  and  $(13111, 0)$



You can read the answers off the graph as  $x = \$7,000$  and  $y = \$5,000$ .

MCAS Open-Response Questions & Answers  
Mathematics -- High School Level

Question #13

- 1) Which of the following pairs of equations and inequalities are equivalent? Explain your reasoning in each case. Site the name of the property used.
- a)  $3x + 2 = 5$                       *and*                       $3x = 3$
- b)  $7x - 8 = 12 + 3x$                       *and*                       $4x = 20$
- c)  $\frac{1}{3}x + 9 = 6$                       *and*                       $x + 9 = 18$
- d)  $10x + 15 = 35$                       *and*                       $2x + 3 = 7$
- e)  $10x + 15 = 35$                       *and*                       $10x = 20$
- f)  $3x + 2 < 5$                       *and*                       $3x < 3$
- g)  $7x - 8 > 12 + 3x$                       *and*                       $4x = 20$
- h)  $10x + 15 < 35$                       *and*                       $-2x - 3 < -7$
- i)  $10x + 15 > 35$                       *and*                       $10x > 20$
- 2) Look back over the pairs of equations and your answers to the equivalence question. What operations on equations and inequalities seem likely to produce simpler equivalent forms?
- 3) What overall strategy and specific reasoning steps would you use to solve an equation of the form  $a + bx = c + dx$ ? Explain how you could check the solution.
- 4) What overall strategy and specific reasoning steps would you use to solve an equation of the form  $a + bx < c + dx$ ? Explain how you could check the solution.
- 5) How do the graphs of expressions like  $y = a + bx$  and  $y = c + dx$  illustrate solutions to the equations and inequalities expressed in number 3 and 4? How would those solutions appear in tables of values for the two functions?

## MCAS Open-Response Questions & Answers Mathematics -- High School Level



### Answers Question #13

- 1) a). Equivalent: Subtracting 2 from each side of the first equation will result in the second equation. (*Addition Property of Equality*)
- b) Equivalent: Subtracting  $3x$  from each side of the equation  
(*Addition Property of Equality*)  
Adding 8 to each side of the resulting equation  
(*Addition Property of Equality*)
- c) Not equivalent: Multiplying the first equation by 3 will result in  $x + 27 = 18$ . (*Multiplication Property of Equality*)
- d) Equivalent: Dividing the first equation by 5 will result in the second equation. (*Multiplication Property of Equality*)
- e) Equivalent: Subtracting 15 from each side of the first equation will result in the second equations. (*Addition Property of Equality*)
- f) Equivalent: Subtracting 2 from each side of the first equation will result in the second equation. (*Addition Property of Equality*)
- g) Equivalent: Subtracting  $3x$  from each side of the equation  
(*Addition Property of Equality*)  
Adding 8 to each side of the resulting equation  
(*Addition Property of Equality*)
- h) Not Equivalent: Dividing the first equation by  $-5$  will require a change in the direction of the inequality. (*Multiplication Property of Inequalities*)
- i) Equivalent: Subtracting 15 from each side of the first equation will result in the second equations. (*Addition Property of Equality*)

## MCAS Open-Response Questions & Answers

### Mathematics -- High School Level

#### *Question 13 continued*

- 2) There are three basic properties:
- ▶ Combining like terms and collecting the variable on one side of the equation or inequality.
  - ▶ Multiplying or dividing both sides by the same number (reversing the inequality sign when multiplying or dividing by a negative number).
  - ▶ Adding or subtracting the same number or expression on both sides.
- 3) Solve the equation by adding equal expression to both sides, which will result in putting the variables on one side of the equation and the constants on the other side, and then multiplying both sides of the equation by the reciprocal of the coefficient of the variable.  
To check, simply substitute the resulting value into the original equations.
- 4) Solve the equation by adding equal expression to both sides, which will result in putting the variables on one side of the equation and the constants on the other side, and then multiplying both sides of the equation by the reciprocal of the coefficient of the variable and reversing the inequality sign if the coefficient is negative.  
To check, simply substitute the resulting value into the original equations.
- 5) When you graph the two equations, you should see lines intersect at the point at which the  $x$  value is your answer to question 3. In the table of values, the function values for that particular value of  $x$  are the same.

For the inequality, you will see one line below the other for your solution to question 4. In the table, the one set of function values will always be less than the other for the  $x$  values corresponding to your solution to the question 4.

# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level

### Question #14

Write an the equations for the lines that are perpendicular to and parallel to the given line through the given point.

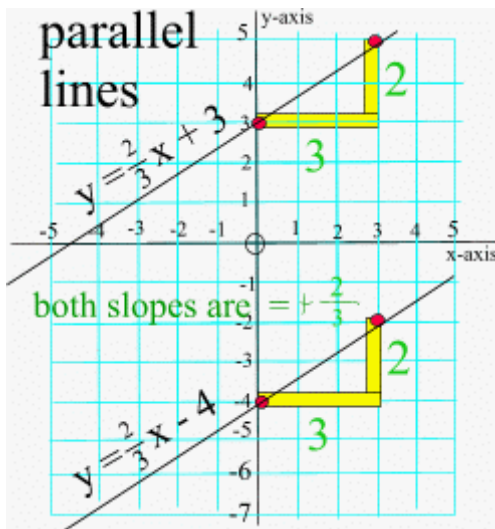
- 1)  $y = 4x - 1$  through  $(-2, -5)$
- 2)  $y = 2x - 3$  through  $(3, -1)$
- 3)  $7x + 2y = 9$  through  $(-7, 2)$
- 4)  $y - 2 = -\frac{2}{3}(x + 1)$  through  $(3, -2)$



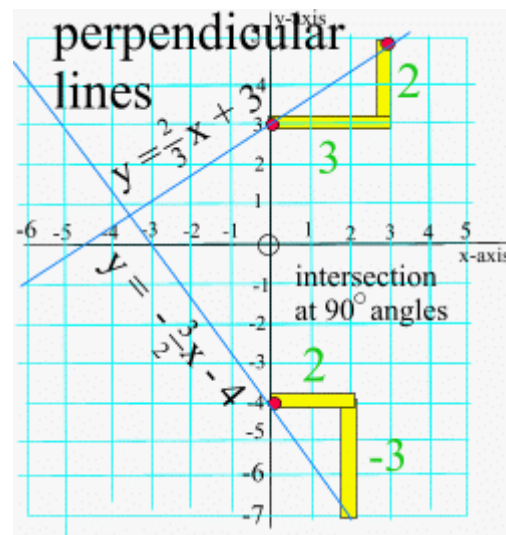
### Answers

#### Question #13

Parallel lines have the same slope.



Perpendicular lines have slopes that are negative reciprocals of each other



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 14 continued

#### Point-Slope Form of an Equation

A line going through the point  $(x_1, y_1)$  and having a slope of  $m$  would have the equation:  $y - y_1 = m(x - x_1)$

a)  $y = 4x - 1$  through  $(-2, -5)$

parallel

$$m = 4$$

$$y - y_1 = m(x - x_1)$$

$$y - (-5) = 4(x - (-2))$$

$$y + 5 = 4(x + 2)$$

perpendicular

$$m = -\frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-5) = -\frac{1}{4}(x - (-2))$$

$$y + 5 = -\frac{1}{4}(x + 2)$$

b)  $y = 2x - 3$  through  $(3, -1)$

parallel

$$m = 2$$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = 2(x - (3))$$

$$y + 1 = 2(x - 3)$$

perpendicular

$$m = -\frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -\frac{1}{2}(x - (3))$$

$$y + 1 = -\frac{1}{2}(x - 3)$$

MCAS Open-Response Questions & Answers  
Mathematics -- High School Level

*Question 14 continued*

c)  $7x + 2y = 9$  through  $(-7, 2)$

$$7x + 2y = 9$$

$$2y = -7x + 9$$

$$y = -\frac{7}{2}x + \frac{9}{2}$$

parallel

$$m = -\frac{7}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - (2) = -\frac{7}{2}(x - (-7))$$

$$y - 2 = -\frac{7}{2}(x + 7)$$

perpendicular

$$m = \frac{2}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - (2) = \frac{2}{7}(x - (-7))$$

$$y - 2 = \frac{2}{7}(x + 7)$$

d)  $y - 2 = -\frac{2}{3}(x + 1)$  through  $(3, -2)$

parallel

$$m = 2 - \frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -\frac{2}{3}(x - (3))$$

$$y + 2 = -\frac{2}{3}(x - 3)$$

perpendicular

$$m = -\frac{3}{2}$$

$$y - y_1 = m(x - x_1)$$

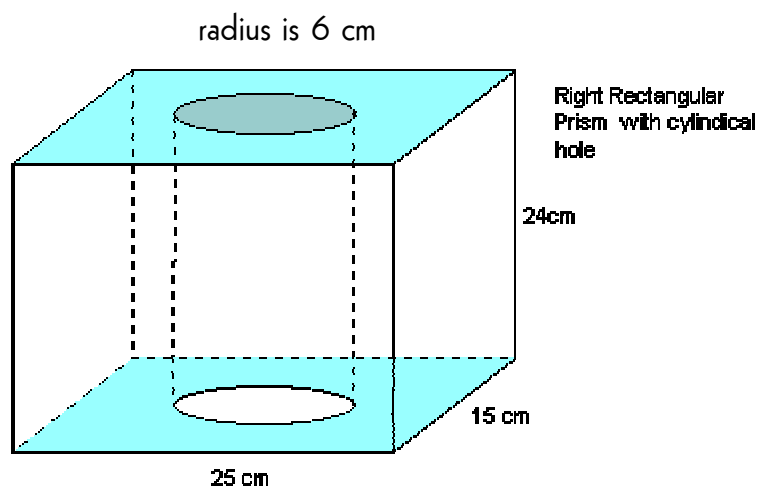
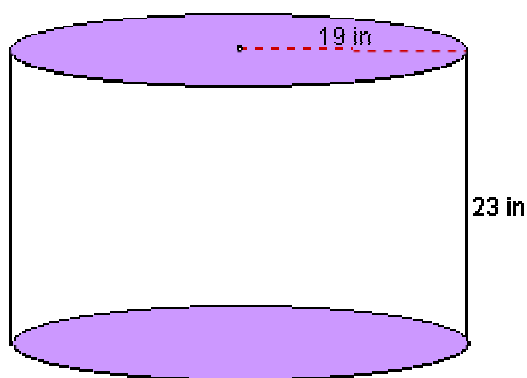
$$y - (-2) = -\frac{3}{2}(x - (3))$$

$$y + 2 = -\frac{3}{2}(x - 3)$$

MCAS Open-Response Questions & Answers  
Mathematics -- High School Level

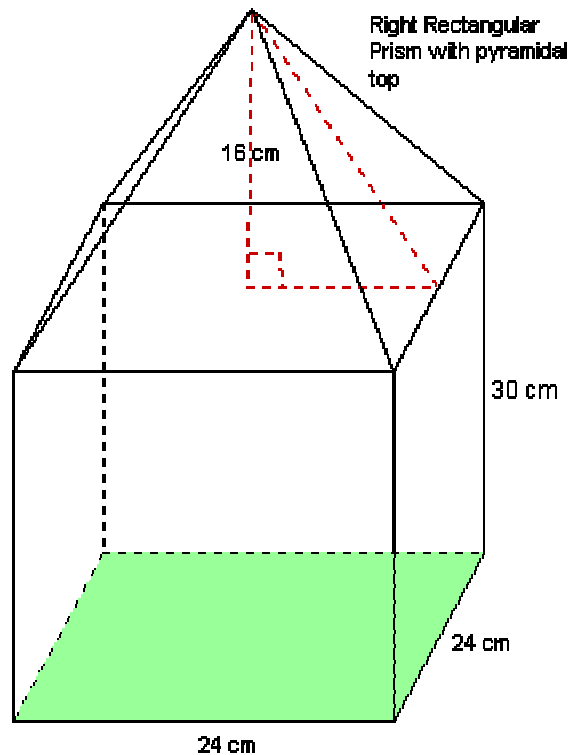
Question #15

Find the lateral area, surface area, and volume of each of the following shapes:



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 15 continued



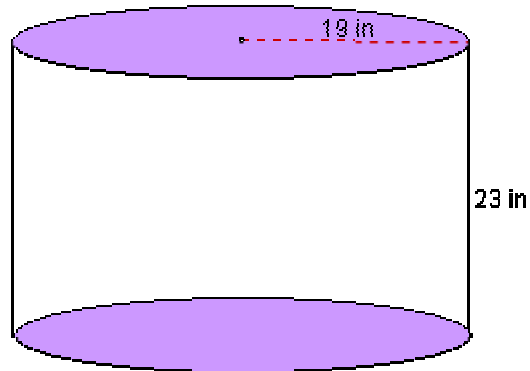
### Answers Question #15

#### Definitions:

- ◆ **Faces** — Sides of a prism.
- ◆ **Bases** — Two parallel congruent polygons, also known as "faces," lying in parallel planes.
- ◆ **Lateral Faces** — The faces that are not bases.
- ◆ **Surface area** — The sum of the areas of all the faces of a prism.
- ◆ **Lateral Area** — The area of all the lateral faces of a prism.
- ◆ **Volume** — The number of cubic units that can be contained in a solid.

MCAS Open-Response Questions & Answers  
Mathematics -- High School Level

Question 15 continued



**Lateral Area:**

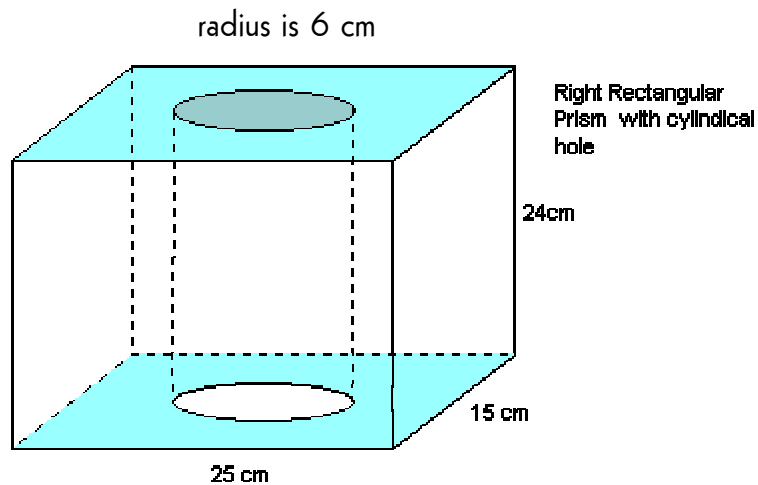
$$\begin{aligned}L &= 2\pi rh \\ &= 2\pi(19)(23) \\ &= 874\pi \text{ in}^2\end{aligned}$$

**Surface Area:**

$$\begin{aligned}S &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(19)(23) + 2\pi(19)^2 \\ &= 874\pi + 722\pi \\ &= 1596\pi \text{ in}^2\end{aligned}$$

**Volume:**

$$\begin{aligned}V &= \pi r^2 h \\ &= (19)^2 (23) \\ &= 8303\pi \text{ in}^3\end{aligned}$$



# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level

### Question 15 continued

#### Lateral Area:

$$\begin{aligned}L_{cube} &= 2lh + 2wh \\ &= 2(25)(24) + 2(15)(24) \\ &= 1920 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}L_{cylinder} &= 2\pi rh \\ &= 2\pi(6)(24) \\ &= 288\pi \text{ cm}^2\end{aligned}$$

$$\begin{aligned}L_{total} &= 1920 + 288\pi \\ &= 2824.779 \text{ cm}^2\end{aligned}$$

#### Surface Area:

$$\begin{aligned}Area_{bases} &= 2(lr - \pi r^2) \\ &= 2((25)(15) - \pi(6^2)) \\ &= 2(375 - 36\pi) \\ &= 523.805 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}S &= 2824.779 + 523.805 \\ &= 3348.584 \text{ cm}^2\end{aligned}$$

#### Volume:

$$\begin{aligned}V_{cube} &= lwh \\ &= (25)(15)(24) \\ &= 9000 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}V_{cylinder} &= \pi r^2 h \\ &= \pi(6)^2(24) \\ &= 864\pi \text{ cm}^3\end{aligned}$$

$$\begin{aligned}V &= 9000 - 864\pi \\ &= 6285.664 \text{ cm}^3\end{aligned}$$



# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level

### Question 15 continued

#### Lateral Area:

$$\begin{aligned}
 L_{\text{pyramid}} &= \frac{1}{2} (\text{slant height}) (\text{perimeter}) \\
 &= \frac{1}{2} (20) (24 \cdot 4) \\
 &= 960
 \end{aligned}$$

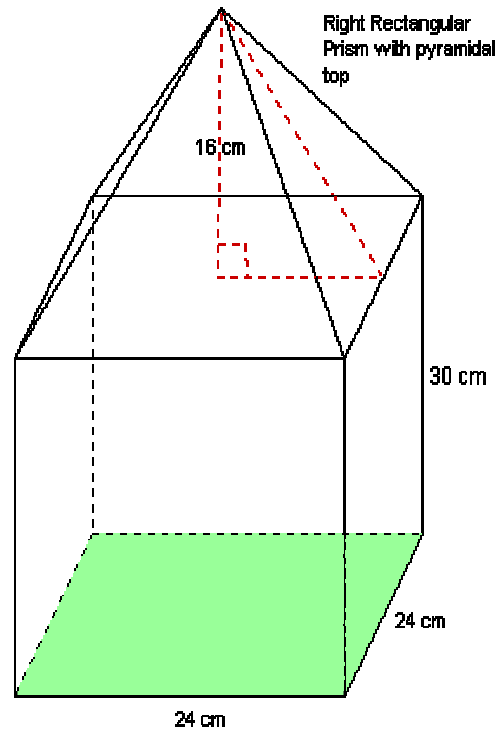
$$\begin{aligned}
 L_{\text{cube}} &= 2lh + 2wh \\
 &= 2(24)(30) + 2(24)(30) \\
 &= 2880 \text{ cm}^2
 \end{aligned}$$

#### Surface Area:

$$\begin{aligned}
 \text{Area}_{\text{base}} &= lw \\
 &= (24)(24) \\
 &= 576 \text{ cm}^2
 \end{aligned}$$

#### Volume:

$$\begin{aligned}
 V_{\text{cube}} &= lwh \\
 &= (24)(24)(30) \\
 &= 17280 \text{ cm}^3
 \end{aligned}$$



$$\begin{aligned}
 L_{\text{total}} &= 960 + 2880 \\
 &= 3840 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 S &= 576 + 3840 \\
 &= 4416 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 V_{\text{pyramid}} &= \frac{1}{3} (\text{Base Area}) h \\
 &= \frac{1}{3} (24^2) (16) \\
 &= 3072 \text{ cm}^3
 \end{aligned}$$

## MCAS Open-Response Questions & Answers

### Mathematics -- High School Level

#### Question 16

If a circle of radius equal to 1.5 units is inscribed in a square such that one point on each side of the square is tangent to the circle, what is the area of the square NOT within the circle?

#### Answer 16

1.93 square units.

#### Question 17

- If  $6 + 3(2x-4) = 12$ , then  $2x-4=?$ , and  $x=?$
- The freshman class is selling book covers for \$0.75 each. If the company has a base cover design cost of \$50, and a printing price of \$0.05 per cover, how many must the class sell to make a \$200 profit?
- At an airport terminal,  $\frac{2}{3}$  of the passengers on an in-coming flight departed the plane and 16 next passengers boarded for the next flight. If there were only 41 passengers on the plane for the second flight, how many passengers were on the incoming flight? Develop an algebraic expression and then solve.

#### Answer 17

- (D)  $2x - 4 = 2$  and  $x = 3$   
(E) 358 book covers need to be sold to make \$200 profit  
(F)  $\frac{1}{3}x + 16 = 41$  where  $x =$  number of passengers on incoming flight,  
 $x = 75$ .



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 18

- a) What is the mathematical expression that can be used to calculate the number of different handshakes possible of a group of "n" people?
- b) The perimeter of a 3:4:5 triangle is 60 inches. What is the length of the hypotenuse? If that leg now became the shortest leg of a 5:12:13 right triangle, what would be the perimeter of that triangle?

### Answer 18

- a)  $n(n - 1)/ 2$
- b) length of hypotenuse = 25 inches. The perimeter of the 5:12:13 triangle would be = 150 inches.

### Question 19

On the weather the other night, the weatherman stated that if a driveway was 10 feet wide and 25 feet long was covered with 1 foot of snow, the snow would weight about 1 ton. Using that information, was is the density of snow in grams/ cubic centimeter?

### Answer 19

The density would be 0.13 gm/cc.

These problems should help with word problems and conversions from one set of measurement units to another.



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 20

Find the area of a triangle with sides measuring 48 cm, 55 cm, and 73 cm.  
An isosceles triangle is situated within a circle such that the base is a cord of length 10 cm, its height is 10 cm, and its vertex angle is at the center of the circle. What is the diameter and circumference of the circle?

### Answer 20

1320 sq. cm.  
Diameter:  $10\sqrt{5}$ , Circumference:  $10\sqrt{5}\pi$

### Question 21

The garden has an area of 1050 square feet. The length of the garden is 5 feet shorter than five times the width. What are the dimensions of the garden?

### Answer 21

70 feet x 15 feet



## MCAS Open-Response Questions & Answers

### Mathematics -- High School Level

#### Question 22

- a) Urban High School plans to maintain one car for driver's education. Each year the value of the car depreciates 25% based on its current value. When the value of the car drops below \$3000, the school buys a new car at the end of the year. The current car cost \$9,200 and after 4 years the value has dropped below \$3000, reaching a value of \$2910.94. The school is going to purchase a new car for \$11,800. Using the same depreciation rate, how many years will it take for the value of the car to drop below \$3000?
- b) 60 % of the cars owned by Trash-It Rental are blue and 70 % have automatic transmission. If you randomly choose a rental car, what is the probability that you will get a blue car with standard transmission?

#### Answer 22

- a) 5 years, value will be \$2800.19
- b)  $\frac{18}{100}$



# Angles & Slides

GRADE SPAN:

LEARNING STANDARD:

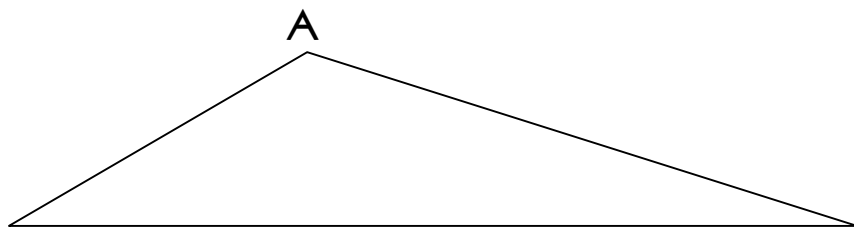
OUTCOME:

- Apply triangle inequality and other inequalities associated with triangles (the longest side is opposite the greatest angle) to prove theorems and solve problems.

## Question #1

In triangle  $ABC$  below, prove that if angle  $A$  has a measure greater than that of angle  $B$ , then  $BC > AC$ .

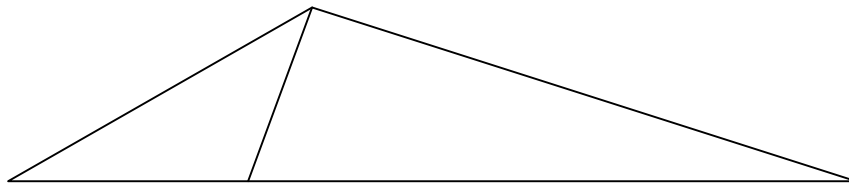
*This idea can easily be extended to show that the side with the greatest length is opposite the angle with the greatest measure.*



## MCAS Open-Response Questions & Answers Mathematics -- High School Level



If the measure of angle **A** is greater than the measure of angle **B**, then by the definition of “greater than” for angles, there exists a point **D** on the line segment **BC** between **B** and **C** such that the measure of angle **B, A, D** is equal to the measure of angle **B**.



By the converse of the Isosceles Triangle Theorem which states that congruent bases imply an isosceles triangle,  $\triangle B, A, D$  is an isosceles triangle with  $AD = BD$ .

By the Triangle Inequality Theorem, we know that the sum of the lengths of two sides of any triangle is always greater than the length of the third side.

Therefore,  $DC + AD > AC$

Since  $AD = BD$ , by the Transitive Property of Equality,  $DC + BD > AC$

Since **D** is on the line segment **BC** between **B** and **C**,  $DC + BD = BC$

Therefore,  $BC > AC$

MCAS Open-Response Questions & Answers  
Mathematics -- High School Level

# Functions

GRADE SPAN:

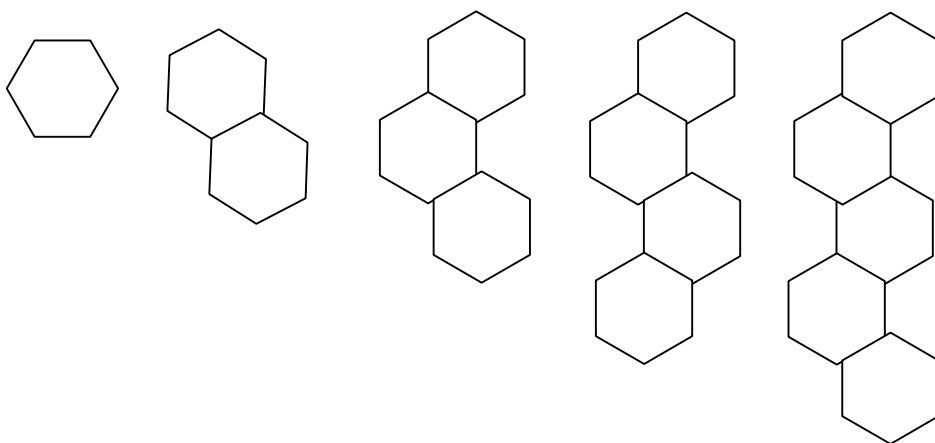
LEARNING STANDARD:

OUTCOME:

- Demonstrate an understanding of relationships and functions. Identify the domain, range, dependent, and independent variables as functions.

## Question #1

- 1) Complete the table based on the pattern below.  
(Can be done with any polygon pattern.)



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 1 continued

Polygon type							
Number of polygons: $n$	1	2	3	4	5	6	7
Perimeter of chain: $P(n)$							

- ▶ Using the pattern in the preceding table, predict the perimeter of a 10-chain figure. Check your answer by drawing it.

$$\text{Perimeter, } P(10) = \underline{\hspace{2cm}}$$

- ▶ Describe in words how you would determine an expression that would tell how to find any term in the sequence of perimeters that you might want to know— no matter how small, no matter how large.
- ▶ Write an expression for finding the perimeter in terms of  $n$  after adding the  $n^{\text{th}}$  polygon.

$$n^{\text{th}} \text{ perimeter is } \underline{\hspace{2cm}}$$

- ▶ Is the relationship between the number of polygons and the perimeter of the chain a function? Why or why not? If so, what does the independent variable represent? the dependent variable?
- ▶ Express the perimeter  $P$  as a function of the number of polygons,  $n$ . What are the domain and range of this function?

$$P(n) = \underline{\hspace{2cm}}$$

$$\text{dom } P = \underline{\hspace{2cm}}$$

$$\text{ran } P = \underline{\hspace{2cm}}$$

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 1 continued

- 2) Draw a scatter plot of the first 7 data points for the function on the TI-82 calculator by following the instructions below.
- ➔ Press **STAT** **EDIT** **1:Edit**
  - ➔ Clear any data that may be in lists **L1** and **L2** by using the up arrow key to move the cursor to highlight the list name, and pressing **CLEAR** followed by **ENTER**.
  - ➔ Enter in **L1** the number of polygons for each entry in your table. Enter the corresponding perimeters (in the same order) in **L2** .
  - ➔ Press **2nd** **STAT PLOT** **1:Plot1**
  - ➔ Select **On**, the first option by **Type:** for a scatter plot, **Xlist:** **L1**, **Ylist:** **L2** and **Mark:**°. To do this, in each case use the arrow keys to move the cursor to the selection and press **ENTER** to select it. After it is selected, it remains highlighted.
  - ➔ Press **ZOOM** **9:ZoomStat** to graph the scatter plot.  
(*This automatically sets the window ranges for the axes to match your data.*)

Describe the scatter plot of the data points verbally.

How do you describe this kind of relationship algebraically?



## MCAS Open-Response Questions & Answers

### Mathematics -- High School Level

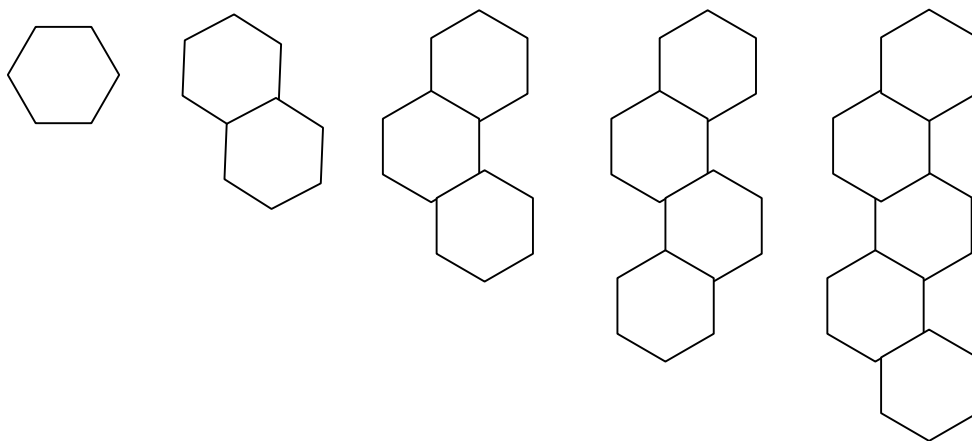


#### Answer Question #1

#### Terminology:

- ➔ A **function**  $f$  from a set **A** to a set **B** is a correspondence between elements of the two sets that matches each element of **A** with **exactly one** element in **B**.
- ➔ The elements of **A**, the input values for the function, comprise the **domain of  $f$** , abbreviated **dom  $f$** . Any element in **B** that corresponds to some element of **A** is called an output value. The set of all output values is called the **range of  $f$** , abbreviated **ran  $f$** .
- ➔ To indicate that a function  $f$  assigns the element  $x$  in **A** to the element  $y$  in **B**, the notation  $f(x) = y$  is used. This is read “the value of  $f$  at  $x$  is  $y$ ”; or, more simply, “ $f$  of  $x$  is  $y$ .”

Complete the table based on the pattern below.



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 1 continued

Polygon type							
Number of polygons: $n$	1	2	3	4	5	6	7
Perimeter of chain: $P(n)$	6	10	14	18	22	26	30

- ➔ Using the pattern in the preceding table, predict the perimeter of a 10-chain figure. Check your answer by drawing it.

$$\text{Perimeter, } P(10) = \underline{\quad 42 \quad}$$

- ➔ Describe in words how you would determine an expression that would tell how to find any term in the sequence of perimeters that you might want to know— no matter how small, no matter how large.

*Try to find a relationship between the number of polygons and the perimeter.  
Look for a pattern between the two sets of numbers.*

- ➔ Write an expression for finding the perimeter in terms of  $n$  after adding the  $n^{\text{th}}$  polygon.

$$n^{\text{th}} \text{ perimeter is } 2 + 4n$$

- ➔ Is the relationship between the number of polygons and the perimeter of the chain a function? Why or why not? If so, what does the independent variable represent? the dependent variable?

*Yes, the relationship between the two is a function because the each perimeter is unique to the number of polygons in the chain.*

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 1 continued

Since the perimeter depends on the number of polygons in the chain, the perimeter is the dependent variable and the number of polygons is the independent variable.

- ➔ Express the perimeter  $P$  as a function of the number of polygons,  $n$ .
- ➔ What are the domain and range of this function?

$$P(n) = 2 + 4n$$

$$\text{dom } P = \text{whole numbers}$$

$$\text{ran } P = \text{whole numbers}$$



# Linear Equations

GRADE SPAN:

LEARNING STANDARD:

OUTCOME:

- Demonstrate an understanding of systems of linear equations that contain two variables.

*Flip Side*

The Twist of Fate

$$\begin{cases} y = -3x + 1 \\ -2x + 2y = -14 \end{cases}$$

- 1) What form is the first equation in?
- 2) What form is the second equation in?
- 3) Convert the second equation into slope-intercept form.

# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level

### *Twist of Fate continued*

#### The Obligatory Table

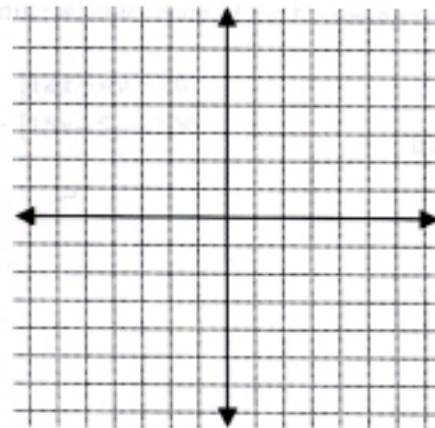
Make a table of values for each equation on the interval from  $[-3, 3]$

X	$y_1$	$y_2$

Why consider the interval from  $[-3, 3]$ ? Why not the interval from  $[-1, 7]$ ?

#### Gotta Love Graphs

Graph the system of linear equations.



# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level

### Question #1

1) Tell if the given point is a solution to the system.

$$(6,9) \begin{cases} y = 4x - 15 \\ y = \frac{1}{2}x - 3 \end{cases}$$

$$(3,8) \begin{cases} y = 4x - 4 \\ y = \frac{2}{3}x + 6 \end{cases}$$

2) Complete the following chart:

Graph of the System	Name of Lines	Number of Solutions	Slopes of the Lines	Y-intercepts of the Lines	Ratios
Two intersecting lines					
Two parallel and nonintersecting lines					
One line (parallel and intersecting)					



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 1 continued

- 3) Determine whether each of the following systems has one solution, infinite solutions, or no solution.

$$\begin{array}{lll} \text{a) } \begin{cases} 12x - 4y = 80 \\ 15x - 5y = 100 \end{cases} & \text{b) } \begin{cases} 2a + 4c = 26 \\ 3a + 6c = 42 \end{cases} & \text{c) } \begin{cases} 2x + 6y = -2 \\ 5x - 3y = 31 \end{cases} \end{array}$$



### Answer Question #1

#### The Twist of Fate

$$\begin{cases} y = -3x + 1 \\ -2x + 2y = -14 \end{cases}$$

- 1) What form is the first equation in?  
**slope-intersect form**
- 2) What form is the second equation in?  
**standard form**
- 3) Convert the second equation into slope-intercept form.

$$y = x - 7$$

# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level

### Question 1 continued

#### The Obligatory Table

Make a table of values for each equation on the interval from  $[-3, 3]$

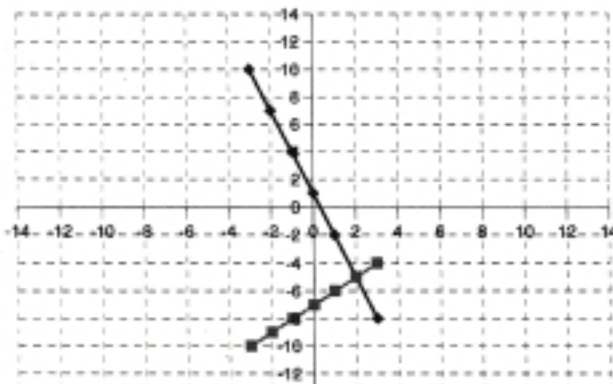
X	$Y_1$	$Y_2$
-3	10	-10
-2	7	-9
-1	4	-8
0	1	-7
1	-2	-6
2	-5	-5
3	-8	-4

Why consider the interval from  $[-3, 3]$ ? Why not the interval from  $[-1, 7]$ ?

symmetric  
interval around  
the origin

#### Gotta Love Graphs

Graph the system of linear equations.



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 1 continued

- 1) Tell if the given point is a solution to the system.

$$(6,9) \begin{cases} y = 4x - 15 \\ y = \frac{1}{2}x - 3 \end{cases} \quad \text{No ... fails second equation} \quad (3,8) \begin{cases} y = 4x - 4 \\ y = \frac{2}{3}x + 6 \end{cases} \quad \text{Yes}$$

- 2) Complete the following chart:

Graph of the System	Name of Lines	Number of Solutions	Slopes of the Lines	Y-intercepts of the Lines	Ratios
Two intersecting lines	<u>Intersecting</u>	<u>One</u>	<u>Different</u>	<u>Different of the same</u>	<u>All Unequal</u>
Two parallel and nonintersecting lines	<u>Parallel</u>	<u>None</u>	<u>Same</u>	<u>Different</u>	<u>Variable ratios are equal</u>
One line (parallel and intersecting)	<u>Coincident</u>	<u>Infinite</u>	<u>Same</u>	<u>Same</u>	<u>All Equal</u>

- 3) Determine whether each of the following systems has one solution, infinite solutions, or no solution.

a) 
$$\begin{cases} 12x - 4y = 80 \\ 15x - 5y = 100 \end{cases}$$

All ratios are equal to  $\frac{4}{5}$  ... coincident lines

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 1:3 continued*

$$\text{b) } \begin{cases} 2a + 4c = 26 \\ 3a + 6c = 42 \end{cases}$$

Variable ratios are equal to  $\frac{2}{3}$  ... parallel lines

$$\text{c) } \begin{cases} 2x + 6y = -2 \\ 5x - 3y = 31 \end{cases}$$

All ratios are unequal ... intersecting lines



# Everyday Problems

GRADE SPAN:

LEARNING STANDARD:

OUTCOME:

- Solve everyday problems (mixture, rate, and work) that can be modeled using systems of linear equations or inequalities and apply algebraic and graphical methods to the solution. Utilize technology when appropriate.

## Question #1

# Health Clubs

You are trying to decide which of two health clubs to join. The first club, "Bust a Gut," charges a fee of \$500 to join, plus a monthly membership fee of \$75. The second club, "Abs & More," charges \$750 to join, plus a \$50 monthly membership fee.



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 1 continued*

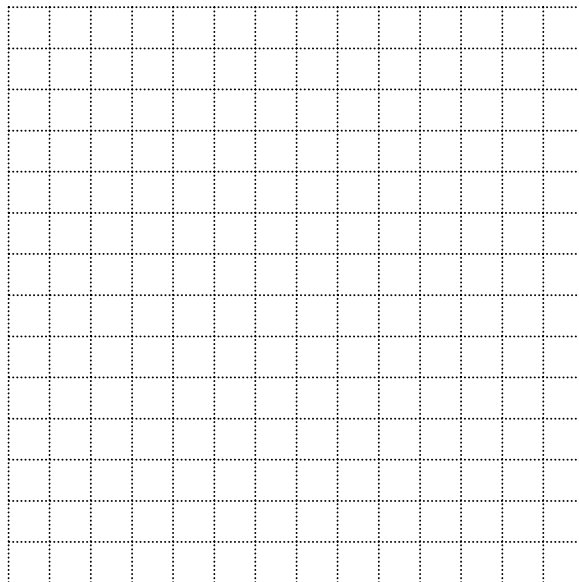
- 1) Complete the table showing the relationship.

Months	"Bust a Gut"	"Abs & More"
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 1 continued*

- 2) The independent variable goes on the horizontal axis, and the dependent variable goes on the vertical axis. Does the length of membership (in months) depend on the cost, or does the cost depend on the length of membership?
- 3) Which variable goes on the horizontal axis?
- 4) On the graph:
  - Number and label axes, and give the graph a title.
  - Plot the points for “Bust a Gut” and join them with a straight line.
  - Plot the points for “Abs & More” and join them with a straight line.
- 5) Estimate from the graph – How long do you have to have a membership before “Abs & More” becomes cheaper?



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### *Question 1 continued*

- 6) What does the slope represent for this problem?
- 7) Find the slope of the line for “Bust a Gut.” Show any work.
- 8) What is the y-intercept of the graph for “Bust a Gut?”
- 9) Find the slope of the line for “Abs & More.” Show any work.
- 10) What is the y-intercept for the graph for “Abs & More?”
- 11) Find the linear equation for:
  - “Bust a Gut”
  - “Abs & More”
- 12) Can you determine exactly how long you have to be a member before “Abs & More” is cheaper?



## MCAS Open-Response Questions & Answers Mathematics -- High School Level



### Answer Question #1

You are trying to decide which of two health clubs to join. The first club, "Bust a Gut," charges a fee of \$500 to join, plus a monthly membership fee of \$75. The second club, "Abs & More," charges \$750 to join, plus a \$50 monthly membership fee.

- 1) Complete the following table showing the relationship.

Months	"Bust a Gut"	"Abs & More"
1	<i>575</i>	<i>800</i>
2	<i>650</i>	<i>850</i>
3	<i>725</i>	<i>900</i>
4	<i>800</i>	<i>950</i>
5	<i>875</i>	<i>1000</i>
6	<i>950</i>	<i>1050</i>
7	<i>1025</i>	<i>1100</i>
8	<i>1100</i>	<i>1150</i>
9	<i>1175</i>	<i>1200</i>
10	<i>1250</i>	<i>1250</i>
11	<i>1325</i>	<i>1300</i>
12	<i>1400</i>	<i>1350</i>

## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 1 continued

- 2) The independent variable goes on the horizontal axis, and the dependent variable goes on the vertical axis. Does the length of membership (in months) depend on the cost, or does the cost depend on the length of membership?

cost depends on length of membership

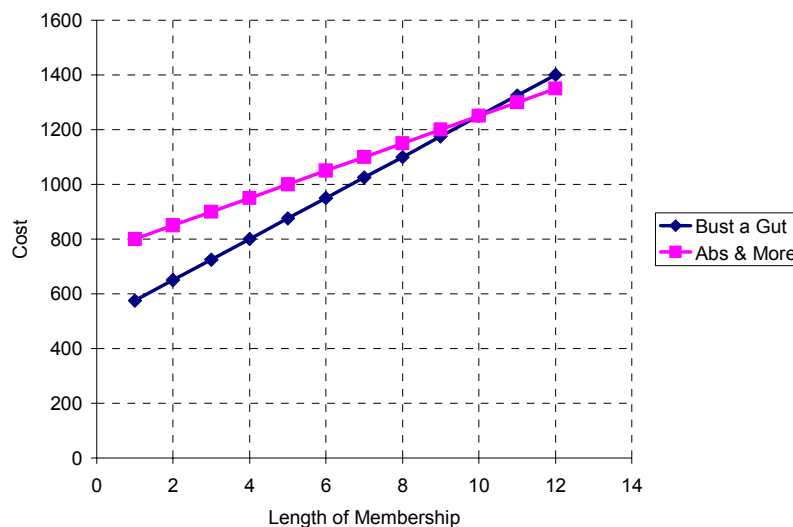
- 3) Which variable goes on the horizontal axis?

length of membership

- 4) On the graph:

- Number and label the axes, and give the graph a title.
- Plot the points for “Bust a Gut” and join them with a straight line.
- Plot the points for “Abs & More” and join them with a straight line.

- 5) Estimate from the graph – How long do you have to have a membership before “Abs & More” becomes cheaper?



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question 1 continued

- 6) What does the slope represent for this problem?  
*change in the price of membership per month*
- 7) Find the slope of the line for "Bust a Gut." Show any work.  
*75 dollars per month*
- 8) What is the y-intercept of the graph for "Bust a Gut?"  
*500 dollars*
- 9) Find the slope of the line for "Abs & More." Show any work.  
*50 dollars per month*
- 10) What is the y-intercept of the graph for "Abs & More?"  
*750 dollars*
- 11) Find the linear equation for:
- "Bust a Gut"  $price = 500 + 75 * months$
  - "Abs & More"  $price = 750 + 50 * months$
- 12) Can you determine exactly how long you have to be a member before "Abs & More" is cheaper?  
*10 months*



## MCAS Open-Response Questions & Answers Mathematics -- High School Level

### Question #2

Mrs. Fuentes wants to buy at least 10 books. Each paperback book costs an average of \$10, and each hardcover book costs an average of \$20. Mrs. Fuentes is planning to spend less than \$250.

- Write a system of linear inequalities that represents this situation.  
Graph the solution to the system.
- Based on your graph, name three possible combinations of paperback and hardcover books that Mrs. Fuentes could buy.



### Answer Question #2

- The variables for this situation would be the number of each type of book:  
 $x = \text{number of paperbacks}$   
 $y = \text{number of hardcover}$

The system therefore would be:

$$\begin{aligned}x + y &\geq 10 \\ 10x + 20y &\leq 250\end{aligned}$$

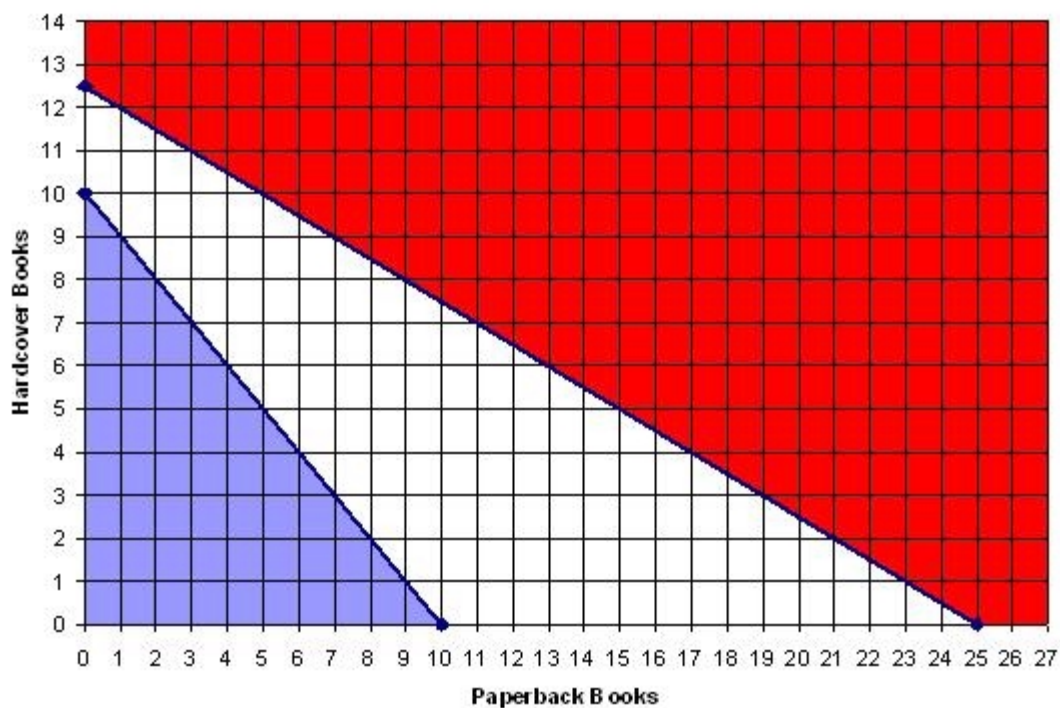


# MCAS Open-Response Questions & Answers

## Mathematics -- High School Level

### Question 2 continued

With a graph of:



The clear section represents the solution set. Any of the coordinate points in this section would represent solutions to the system.